#### **General Theses**

# Self-Compensating Control for Suppression of Rotational Speed Variation Generated in a Geared Servo System

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# 1. Introduction

The merit of using gear reducers in servo-controlled mechanical systems is that the power transfer system can be designed in a compact configuration with high efficiency by using a higher motor rotation speed in comparison with a direct-drive motor. In practice, however, when a reducer is installed in a mechanical system, the required dynamic characteristics of the system may not be achieved due to velocity and torque ripples in the lower frequency range of the bandwidth of the main control system even though the adequate selection is made in accordance with the specifications. Because, velocity and torque ripples occur inside the gear reducer depending upon its structure and the rotational speed of the input shaft, and excite the driven machine system.

To overcome this problem, a gear reducer with self-compensating control function is proposed and its fundamental performance is described in the previous Technical Report (1). In summary, the excitation components that occur in the lower frequency range caused by the structure of the reducer itself, are detected by a pulse generator attached to the final geared stage without reducing the torsional stiffness of the reducer. The ripple component detected by the pulse generator is fed back to the servo amplifier for compensation.

This report clarifies the effectiveness of this technique by investigating the stability, the command following performance and the disturbance rejection performance of the control system, assuming that the self-compensating control is applied to a mechanical system. In addition, this report describes the suppression effect on the rotational speed variations which occur at passing through resonance in the mechanical system composed of the harmonic drive gear reducer <sup>(2) (3)</sup>.

# 2. Internal Exciting Torque Generated inside the Gear Reducer

The equation of motion, which describes the torsional vibration of the mechanical system including the gear reducer, can be expressed by the differential equation assuming that the stiffness of the gearing is constant.

 $M\ddot{X} + C\dot{X} + KX = T_{rq} - --(1)$ 

where M, C, and K represent a mass matrix, a damping matrix and a stiffness matrix, respectively. In addition, Trq represents an external exciting torque vector and X indicates a displacement vector. If an angle transmission error e(t) (error vector) exists at the geared stage, Eq.(1) is rewritten as

$$M\ddot{X} + C\dot{X} + K(X - e) = T_{rq} - --(2)$$

Further, when M is not singular, Eq.(2) is expressed as

Consequently, an internal exciting torque Qd (vector) is locally generated inside the gear reducer.

A typical example of such internal excitation can be found in the harmonic drive gear system. Fig. 1 shows a schematic diagram of the gear head  $^{(2)(3)}$  with a sensor for self-compensating control using the harmonic drive gear reducer. Various researches have been investigated the causes of the internal excitation  $^{(4)}$  to  $^{(7)}$ . Here, the cause of the internal excitation locally generated inside the harmonic drive gear system is assumed from the study of Hidaka  $^{(4)}$  et al. to be the angle transmission error that is produced by the radial alignment errors at the circular spline and flexspline. Denoting the variable component of the angle transmission errors between the input and the output shafts as  $\theta_e$ ,

$$\theta_{r} = \frac{\tan \alpha}{r_{r}} \Big[ \sum_{i=2}^{\infty} A_{ii} \sin(i\theta_{i} + \psi_{ii}) + \sum_{i=2}^{\infty} A_{ii} \sin(i\theta_{i} + \psi_{ii}) \Big]_{\dots} \Big]_{(4)}$$
  
(i=2, 4, 6...)

$$\theta_c = \int \omega_m(t) dt$$
,  $\theta_f = \int (Z_c / Z_f) \omega_m(t) dt$  ---(5)

where,

 $\alpha$  : pressure angle of teeth

*rc* : pitch radius of the circular spline

 $Z_{c}, Z_{f}$  : number of teeth of the circular spline and the flexspline, respectively

*Aci*,*Afi* : i-th amplitude (radial direction error)

 $\psi_{ci}, \psi_{fi}$  : i-th phase angle

Eq. (3) shows that an internal exciting torque that is generated inside the gear reducer, can be expressed by the product of the variable component  $\theta_e$  of the angle transformation error and torsional stiffness  $K_g$  of the reducer. Consequently the variable component Qd(t) of the torque (scalar) is expressed as

$$Q_d(t) = K_t \theta_t$$
  
=  $K_t \Big[ \frac{\tan \alpha}{r_t} \Big\{ \sum_{i=2}^{\infty} A_{ii} \sin (i\theta_t + \psi_{ii}) + \sum_{i=2}^{\infty} A_{ii} \sin (i\theta_t + \psi_{ii}) \Big\} \Big] \dots (6)$   
(i=2, 4, 6...)

Eq. (6) shows that Qd(t) consists of the fundamental component of two cycles per motor revolution and its higher harmonics. In this case, each variation component has a beat frequency, since Zc is nearly equal to Zf.

### 3. Variable Self-Compensating Control

Fig. 2 shows the block diagram of the geared motor system composed of the self-compensating type gear head with the self-compensating control loop that consists of variable center frequency type tracking band-pass filters, and the driven machine system.

<sup>w</sup>*cmd*: velocity command

- $\omega_m$  : rotating speed of the motor (= rotating speed of the wave generator)
- ${}^{\omega}g$  : rotating speed of the gear reducer's output shaft
- $\omega_l$  : rotating speed of the driven machine
- $\theta_m$  : angular rotation of the motor
- $\theta g$  : angular rotation of the gear reducer's output shaft
- $\theta_l$  : angular rotation of the driven machine
- $R_g$  : reduction ratio of the gear reducer
- *i* : current of the armature
- **R** : motor armature resistance
- *L* : motor armature inductance
- $K_t$  : torque constant
- *Ke* : voltage constant
- *Kc* : current loop gain
- *Kcb* : current feedback gain
- $K_v$  : proportional gain of the PI control in the velocity control system
- $T_i$  : integral time constant of the PI control in the velocity control system
- $J_m$  : moment of inertia of the motor
- $J_g$  : moment of inertia of the reducer's output shaft
- $J_l$  : moment of inertia of the driven machine
- $K_g$  : torsional stiffness of the reducer
- $K_s$  : torsional stiffness between the reducer's output shaft and the driven machine
- $C_g$  : damping factor of the reducer
- $C_s$  : damping factor between the reducer's output shaft and the driven machine

Gv(s) represents the transfer function of the PI control in the velocity control system.

 $G_r(s) = K_r(1+1/T_is) - --(7)$ 

G b(s) shows the transfer function of the self-compensating control loop, which is composed of three band-pass filters and feedback gains in parallel, in order to reduce up to the third component of variations (cf., Eq. 6) generated within the bandwidth of the main control loop. Following up the velocity command value, the center frequencies of band-pass filters  $\omega_{oi}$  ( $i = 1 \sim 3$ ) are set to twice, four times and six times of the motor rotating frequency as

$$\begin{array}{c} \omega_{e^{1}}(t) = 4\pi N_{m}(t) / 60 \\ \omega_{e^{2}}(t) = 2\omega_{e^{1}}(t) \\ \omega_{e^{3}}(t) = 3\omega_{e^{1}}(t) \end{array} \right\} ---(8)$$

The center frequencies of tracking band-pass filters  $\omega_{oi}$  (i = 1 ~ 3) are time-varying, while Q factors of the band-pass filters are constant ( $Q_{oi} = \text{const.}$ ). The state equation that indicates the performance of the system composed of the driven mechanical system and the driving system (electrical and mechanical systems), is as follows.

$$dx/dt = Ax + Bu + Eq$$
 ---(9)  
 $y = Cx$  ---(10)

u, q, x and y represent an input variable, a disturbance, a state vector and an output vector, respectively, where  $u = \omega_{cmd}$  and q = Qd(t). As  $\alpha_j$  and  $\beta_j$  ( $j = 1 \sim 3$ ) are the state variables of the band-pass filters, and  $K_{bj}$  ( $j = 1 \sim 3$ ) is the feedback gain of

the self-compensating control loop, and each vector and the coefficient matrixes are expressed as follows:

where,

## 4. Characteristics of Self-Compensating Control System

### 4.1 Stability

When the center frequencies of the tracking band-pass filters change with the change of motor rotation speed, the system stability is considered by calculating the loci of system eigenvalues, assuming that the system matrix A changes slowly with the changes of the center frequencies. Table 1 shows the parameter values that are used for simulations. Fig. 3 shows the loci of system eigenvalues around the imaginary axis when the conditions of the feedback gain Kb 3 of the self-compensating control loop are changed. When Kb 3 = 120 where the feedback gain for the higher-order ripple component has a high gain, the system becomes unstable as the motor axis rotating speed exceeds 2158 min<sup>-1</sup> ( $\omega_{ol} = 452$  rad/sec).

Therefore, when Kb 3 is set at around 120 in order to decrease the higher harmonic component 6Nm in the lower motor speed range, Kb 3 must be lowered in a higher motor speed range to satisfy the stability conditions of the system.

Consequently, considering the stability conditions and compensation effects within the bandwidth which is the objective of this method, Kb 3 is set to zero and the third variation component is not fed back into the current loop in the higher motor speed range.

# **Table 1 Simulation conditions**

Parameter		Value	Unit	
Moment of inertia	$\boldsymbol{J}m$	1.011×10 <sup>-5</sup>	kg•m2	
	Jg	6.565×10 <sup>-5</sup>		
	Jl	8.205×10 <sup>-4</sup>		
Torsional stiffness	Kg	5053.5	N•m/rad	
	Ks	80.984		
Damping coefficient	Cg	0.13	N*m*sec/rad	
	Cs	0.013		
Gear reducer				
Reduction ratio	<b>R</b> g	50.0	-	
Pressure angle	α	30.0	degree	
Pitch diameter	2rc	38.0	mm	
Number of teeth	$\mathbf{Z}c,\mathbf{Z}f$	204,200	-	
Radial error	Ac2,Af2	3.0,2.0	$^{\mu}m$	
	<i>Ac4</i> ,Af4	5.5,5.0		
	Ac6,Af6			
	3.0,2.0			
$\begin{aligned} \psi_{ci}, \psi_{fi} \\ (i = 2, 4, 6) \end{aligned}$	0.0,0.0	degree		
Velocity loop gain	<b>K</b> v	0.3913	A/(rad/sec)	
Integral time constant	<b>T</b> i	0.00563	sec	
Torque constant	<b>K</b> t	0.2633	N•m/A	
Voltage constant	Ke	0.1810	V/(rad/sec)	
Phase resistance	R	35.0	Ω	
Phase inductance	L	0.022	Н	
Current loop gain	<b>K</b> c	3.05	V/A	
Current feedback gain	<b>K</b> cb	1.0	-	
Self-compensating loop feedback gain	<b>K</b> b1	30.0	A/(rad/sec)	
	<b>K</b> b2	60.0		
	<b>K</b> b3	120.0	; if Nm≦1800min <sup>-1</sup>	
		0.0	; else if Nm≧1800min <sup>-1</sup>	
Q factor of band-pass filter	<b>Q</b> 01	12.0	-	
	<b>Q</b> 02	20.0		
	<b>Q</b> 03	25.0		

### 4.2 Command Following Performance

Setting q = 0, the transfer function matrix Gu(s), which represents the command following performance between the velocity command  $\omega_{cmd}$  and the rotating speed, is obtained from Eq.(9) and (10) as

$$G_{w}(s) = [G_{w11} G_{w12}]^{T} = C(sI - A)^{-1}B - --(12)$$

The transfer function  $\omega_g / \omega_{cmd}$  for the reducer's rotation speed of velocity command  $\omega_{cmd}$  is the  $Gu_{12}(s)$  component of Gu(s). Command following performance is investigated when the loading inertia is directly connected to the reducer's output shaft. The mechanical parameters of the driven machine part are as follows:

 $J_l = 2.539 \times 10^{-4} \text{ kg} \cdot \text{m}^2$  $K = 16000.0 \text{ N} \cdot \text{m/rad}$  $C = 0.13 \text{ N} \cdot \text{m} \cdot \text{sec/rad}$ 

The other simulation parameters are given in Table 1. Fig. 4 shows the bode plots (result of simulation) of the transfer function  $\omega_g / \omega_{cmd}$  using the self-compensating control loop set to the motor speed of 750 min<sup>-1</sup>. The reverse triangle mark ( $\nabla$ ) in Fig. 4 represents the internal exciting frequencies (25, 50 and 75 Hz). In Fig. 4, a peak of about 580 Hz noted by the black reverse triangle mark ( $\nabla$ ) represents the first natural frequency of the torsional vibration of the mechanical system. This figure shows that  $G_{u12}(s)$  has band-eliminating characteristics due to the band-pass filters of the self-compensating control loop.

### 4.3 Disturbance Rejection Performance

Setting u = 0, the transfer function matrix Gd(s), which represents the disturbance rejection performance of the self-compensating control loop, is obtained from Eq.(9) and (10) as

 $G_d(s) = [G_{d11} G_{d12}]^T = C(sI - A)^{-1}E - (13)$ 

The transfer function  $\omega_g / Qd$  for the reducer's rotation speed of the internal excitation Qd is the  $Gu_{12}(s)$  component of Gd(s).

Fig. 5 shows the bode plots of the transfer function  $\omega_g / Qd$  when the

self-compensating control loop set for the motor speed of 750 min<sup>-1</sup> is used. This figure shows that the self-compensating control suppresses the speed variations of the reducer generated by the internal exciting components (at 25, 50 and 75 Hz noted by the reverse triangle ( $\bigtriangledown$ ) in Fig. 5) which occur depending upon the rotating speed.

# 5. Simulation on Suppression of Vibration when Passing Through Resonance

Vibration suppression effects when passing through resonance are examined by simulation. Table 1 shows the values of parameters used for the simulations. Values of the radial direction errors (Aci, Afi,  $\psi ci$ ,  $\psi fi$  : i = 2, 4, 6) of the harmonic drive gear reducer are set referring to the reference (4). The first natural frequency of the torsional vibration of the mechanical system is 50 Hz. In this simulation, the time historical response of the rotating speed  $\omega l$  of the driven mechanical system is calculated by the Runge-Kutta method. To avoid excitation of the free vibration in

the driven machine system, the angular acceleration is set to 62.8 rad/sec<sup>2</sup> (converted to motor shaft). This condition means that the motor rotation speed is accelerated or reduced to 3000 min<sup>-1</sup> in five seconds. On the other hand, *Kb3* will be set to zero on the condition of  $N_m \ge 1800 \text{ min}^{-1}$  of the motor speed to satisfy the stability condition. Fig. 6 shows simulation results. This figure shows that the applied variable

self-compensating control suppresses the vibration level of the loading inertia when passing through resonance (Nm = 500, 750 and 1500 min<sup>-1</sup> of the motor speed) by reducing the excitation components locally generated inside the gear reducer.

## 6. Experimental Results and Considerations

### 6.1 Hardware Configuration

Fig. 7 shows a configuration of the geared motor system with the variable self-compensating control system that is configured to reduce velocity ripple generated by internal exciting torque shown by Eq. (6). To detect the velocity ripples that are locally generated inside the gear reducer, a magnetic pulse encoder with 1000 pulses/rev. is equipped with a final geared stage without decreasing the torsional stiffness of the output shaft. The output pulses are converted to an analog signal through the F/V converter with a frequency doubler, and the variable component to be decreased is fed back to the current input of the servo amplifier through band-pass filters.

The tracking band-pass filters that are integrated into the self-compensating control loop are composed of switched capacitor filters. Their center frequencies keep tracking the velocity command generated from the pulse generator and are adjusted to the speed variation components locally generated at the final geared stage. The velocity variation components that are fed back to the current loop of the servo amplifier are limited to the third-order component considering the stability and expecting compensation effects within the bandwidth of the main control loop. In addition, taking the simulation results of sections 4.1 and 5 into account, the feedback gain Kb3 is set to zero by the gain selector and is not fed back to the current loop to stabilize the system when the motor speed Nm  $\geq$  1800 min<sup>-1</sup>.

### 6.2 Suppression Effects on the Rotational Speed Variations under No-loading

Fig. 8 shows effects of the self-compensating control under no-loading at the steady-state motor rotation. The horizontal axis represents the frequency and the vertical axis indicates the velocity ripple at each motor speed. This figure shows that variation components of  $2N_m$ ,  $4N_m$  and  $6N_m$  are reduced down to  $1/3 \sim 1/2$ .

### 6.3 In the Case of Loading Condition

Fig. 9 outlines the experimental set-up with loading inertia. The velocity signal of the loading inertia is detected with an encoder of 81,000 pulses/revolution and it is analyzed after being converted to an analog signal through the F/V converter. The physical parameters of this experimental set-up are shown in Table 2. The first natural frequency of the torsional vibration of the loading mechanical system is 50 Hz.

Fig. 10 shows suppression effects on the vibration level of the loading inertia at the steady-state motor speed. At the motor speeds of 500, 750, and 1500 min<sup>-1</sup> where the variation components of 6Nm, 4Nm and 2Nm resonate at the natural frequency, the speed variation level of the loading inertia is increased. The variable self-compensating control applied to the harmonic drive gear reducer clearly

suppresses the vibration level down to 1/3 to 1/2.

Parameter		Value	Unit
Moment of inertia	Jm	1.011×10 <sup>-5</sup>	kg m2
	$J_{g}$	6.565×10 <sup>-5</sup>	
	Jı	8.205×10 <sup>-4</sup>	
Torsional stiffness	Kg	5053.5	N*m/rad
	Ks	80.984	
Damping coefficient	Cg	0.13,	N*m*sec/rad
	Cs	0.013	
Reduction ratio of gear reducer	Rg	50.0	-

### Table 2 Physical parameters of the experimental set-up

### 6.4 Suppression Effects when Passing Through Resonance

Fig. 11 shows suppression effects of vibration when passing through resonance. The condition of angular acceleration is  $62.8 \text{ rad/sec}^2$  (converted to motor shaft). Fig. 11 shows the suppression effect of about  $1/3 \sim 1/2$  to the uncompensated vibration level in this case, too.

# 7. Conclusion

A variable self-compensating control technique to reduce the speed variation locally generated inside the gear reducer, is applied to the harmonic drive gear reducer. Its fundamental characteristics and installation effects in the servo system were examined theoretically and experimentally. The results of this report are summarized below.

(1) The effects of this self-compensating control system were clarified by simulating the stability, command following characteristics and disturbance rejection performance.

(2) Using the self-compensating control loop, the variation of no-loading inertia can be suppressed down to about  $1/3 \sim 1/2$ . (3) From the simulations and experiments on the system of the loading inertia, it was confirmed that the self-compensating control system can suppress vibration when passing through resonance. The simulation results agreed well with the experiment results qualitatively. The velocity variation can be suppressed down to about  $1/3 \sim 1/2$ .

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Fig.1 Schematic diagram of a gear head with a sensor for self-compensating control



Fig.2 Block diagram of the geared motor system with the variable self-compensating control loop



Fig.3 Loci of system eigenvalues



Fig. 4 Bode plots of  $\omega g/\omega$  cmd









Fig.7 Configuration of the geared motor system with the variable self-compensating control loop





Fig.8 Suppression effects under no-loading

# Fig.9 Configuration of the experimental set-up with loading inertia



Fig.10 Suppression effects on the rotational speed variations in the case with loading inertia



Fig. 11 Suppression effects when passing through resonance (experiment)

