

# Compensation Method for Three-phase AC-DC Converter

## Characteristics on the Condition of Unbalanced AC side Voltage

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### 1. Introduction

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To counter the problem of harmonic currents in the electric power system and to conserve energy, several papers have been published on a three-phase high power factor converter that shapes the input current to an AC-DC converter into a sine wave and improves the power factor to almost 1.<sup>(1)</sup> However, the characteristics of these converters have been analyzed in most cases using the assumption that the three-phase AC input voltage is supplied in the balanced condition.<sup>(2)</sup> If the three-phase high power factor converter is operated while the three-phase AC input voltage is unbalanced, the input current will be distorted and the DC output voltage will include low harmonic frequency components. The three-phase AC input voltage is often unbalanced in practice, and this degrades the characteristics of the three-phase high power factor converter. We report here on a compensation system in which the characteristics of the three-phase high power factor converter are not adversely affected even when the input voltage unbalanced. In this paper, following points are described :

- (1)Characteristics of three-phase high power factor converter with unbalanced input voltage,
- (2)Overview of characteristics compensation circuit
- (3)Verification by simulation and experiment.

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### 2. Characteristics of High Power Factor Converter with Unbalanced AC Input Voltage

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First, we describe how the characteristics of a high power factor converter deteriorate when the three-phase AC input voltage becomes unbalanced using an example of a three-phase high power factor converter with multiple-connected single-phase back type choppers.

[Fig. 1](#) shows the main circuit configuration of this high power factor converter.

<sup>(3)</sup>The respective three-phase input voltages are insulated by transformers and are separated into each phase. The circuits of the respective phases consist of the back type chopper as the base, and are controlled so that the input current,  $i$ , becomes sinusoidal and the power factor becomes 1 by the PWM control. The PWM function,  $F$ , can be expressed by the following equation when the PWM signal amplitude is 1.<sup>(4)</sup>

$$F = M \sin \omega t + F_h \quad \dots (1)$$

where,  $M$ : modulation factor ( $0 < M < 1$ ),

$F_h$ : function caused by the PWM pattern.

The reactor,  $L_{dc}$ , of the output filter functions as the current source, and the reactor current,  $i_L$ , is assigned to the respective phases by the back type chopper of each phase. Therefore, the current,  $i_s$ , that flows through the switch of each chopper can be expressed by the following equation using the PWM function,  $F$ .

$$i_s = |i_L(M \sin \omega t + F_h)| \quad \dots (2)$$

Of the current,  $i_s$ , the component that is related to,  $F_h$ , is supplied from the capacitor,  $C_{ac}$ , that is used in the input. The remaining components become the input current,  $i$ , that is expressed by the following equation:

$$i = i_L M \sin \omega t \quad \dots (3)$$

The above equation shows that the input current,  $i$ , becomes a sine wave when the reactor current,  $i_L$ , is a constant value.

Here, the DC voltage,  $V_{dc}$ , is obtained by connecting each outputs of respective back type choppers, and by filtering them. The output voltage,  $V_f$ , of the back type chopper of each phase can be expressed by the following equation:

$$\begin{aligned} V_f &\doteq |e| \times (M \sin \omega t + F_h) \\ &= \sqrt{2} M E_{ac} \sin^2 \omega t + |e| \times F_h \quad \dots (4) \\ &= \frac{\sqrt{2} M E_{ac} (1 - \cos 2 \omega t)}{2} + |e| \times F_h \end{aligned}$$

where,  $e$ : instantaneous value of AC input voltage

$$(e = \sqrt{2} E_{ac} \sin \omega t)$$

$E_{ac}$ : effective value of AC input voltage.

On the above equation, the output voltage include a low frequency component that is two times the power line frequency. However, the low frequency component is able to cancel using a principle that the sum of the three-phase balanced voltages becomes zero, as expressed by the following equation :

$$\begin{aligned} V_{3f} &= V_{fu} + V_{fv} + V_{fw} \\ &= \frac{\sqrt{2}}{2} \{ M_u E_{acu} + M_v E_{acv} + M_w E_{acw} - M_u E_{acu} \cos 2 \omega t \\ &\quad - M_v E_{acv} \cos 2(\omega t + \frac{2\pi}{3}) - M_w E_{acw} \cos 2(\omega t + \frac{4\pi}{3}) \} \quad \dots (5) \\ &\quad + \{ |e_u| F_{hu} + |e_v| F_{hv} + |e_w| F_{hw} \} \end{aligned}$$

where, the subscripts **u**, **v** and **w** correspond to the **U**, **V** and **W** phases.

When the three-phase AC input voltage is in the balanced condition, the effective value of each phase becomes unity, i.e.,  $E_{ac} = E_{acu} = E_{acc} = E_{acw}$ . Therefore, the modulation factor of each phase becomes equal ( $M = M_u = M_v = M_w$ ), and the synthesized voltage,  $V_{3f}$ , can be expressed by the following equation:

$$V_{3f} = \frac{3ME_{ac}}{\sqrt{2}} + \{ |e_u|F_{hu} + |e_v|F_{hv} + |e_w|F_{hw} \} \quad \text{--- (6)}$$

This synthesized voltage consists of the DC component given by the first term in the right side, and of the high frequency components that are related to switching. Because low frequency components are not included in the synthesized voltage,  $V_{3f}$ , the reactor current,  $iL$ , that flows in the  $L_{dc}$ , does not also include low frequency component. So, since the reactor current,  $iL$ , becomes a DC current that does not include a low frequency component, the input current,  $i$ , is controlled by the sine wave as shown in Eq.(3). In addition, the DC voltage,  $V_{dc}$ , that does not include the low frequency component can be obtained as the following equation because the high frequency component of the synthesized voltage,  $V_{3f}$ , is removed by the smoothing filter consisting of,  $L_{dc}$ , and,  $C_{dc}$ , on the DC side.

$$V_{dc} = \frac{3ME_{ac}}{\sqrt{2}} \quad \text{--- (7)}$$

If the three-phase AC input voltage becomes unbalanced, Eq.(5) cannot be transformed to Eq.(6), and the synthesized voltage,  $V_{3f}$ , includes the low frequency component. Accordingly, the reactor current includes the low frequency component, and the input current is distorted as shown in Eq.(3). Since the smoothing filter in the DC side can remove the high frequency components of the synthesized voltage,  $V_{3f}$ , the low frequency components appear in the DC voltage, too. The waveforms actually measured at the various points are shown in [Fig. 2](#) when the input voltage becomes unbalanced. The unbalanced factor,  $k$ , is 10% and is expressed by the following equation:<sup>(5)</sup>

$$k = \frac{E_2}{E_1} \quad \text{--- (8)}$$

where,  $E_1$ : positive phase sequence component,

$E_2$ : negative phase sequence component.

The input current,  $i_w$ , of Fig. 2 (b) is distorted because the input voltage is unbalanced as shown in Fig. 2 (a). The reactor current,  $i_L$ , includes the low frequency components and the high frequency components caused by switching as shown in Fig. 2 (c), and the DC voltage,  $V_{dc}$ , includes the low frequency ripple as shown in Fig. 2 (d).

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### 3. Compensation Method for Characteristics of High Power Factor Converter for Unbalanced AC Input Voltage

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#### 3.1 Overview of Characteristics Compensation Circuit

As long as the three-phase AC input voltage is in the unbalanced condition, the term,  $\cos 2\omega t$ , in Eq.(5) cannot be removed, and the characteristics of the converter are degraded. If the each modulation factor is varied in accordance with the effective value of the input voltage of each phase as shown in the following equation, the term,  $\cos 2\omega t$ , in Eq.(5) can be removed even though the three-phase AC input voltages are unbalanced.

$$M_u \cdot E_{acu} = M_v \cdot E_{acv} = M_w \cdot E_{acw} = M \cdot \bar{E}_{ac} \quad \dots (9)$$

Because input voltage of each phase is electrically insulated by the transformer on the input side, the respective input currents of the three-phase high power factor converter can be independently controlled. This means that the modulation factor of each phase can be easily varied, and thus the modified modulation factor can be calculated for each phase. Eq.(9) can be transformed to the following equation using an unbalanced rate,  $k'$ , which is the ratio of the average value of the three-phase input voltage to the effective value of each phase.

$$\frac{M}{k'_u} k'_u \bar{E}_{ac} = \frac{M}{k'_v} k'_v \bar{E}_{ac} = \frac{M}{k'_w} k'_w \bar{E}_{ac} = M \bar{E}_{ac} \quad \dots (10)$$

$$k'_u = \frac{E_{acu}}{\bar{E}_{ac}}, \quad k'_v = \frac{E_{acv}}{\bar{E}_{ac}}, \quad k'_w = \frac{E_{acw}}{\bar{E}_{ac}} \quad \dots (11)$$

Therefore, the modulation factor,  $M_{u,v,w}$ , of each phase can be expressed by the following equation:

$$M_u = \frac{M}{k'_u}, \quad M_v = \frac{M}{k'_v}, \quad M_w = \frac{M}{k'_w} \quad \dots (12)$$

The control circuit that implements this compensation method is as follows. [Fig. 3](#) shows the block diagram for the characteristics compensation of the three-phase high power factor converter when the three-phase input voltages are unbalanced. Compensation is performed by the feed-forward control that does not require a current detector. First, the effective value of the input voltage of each phase is obtained by detecting the input voltage of each phase. Using the detector outputs, the average value of the three phases is calculated and the unbalanced rate,  $k'$ , is obtained. The modulation factor of each phase is corrected by the,  $k'$ , hence signal,  $V_{rl}$ , is obtained. The reference signal,  $V_r$ , is obtained by multiplying the signal,  $V_{rl}$ , by the error signal of the DC voltage.

#### 3.2 Enlargement of Compensation Range Using the Third Harmonic Injection Method

This high power factor converter uses the PWM system which compares the sawtooth carrier signal with the reference signal. In the PWM system, the modulation factor becomes 1 when the peak of the carrier signal agrees with that of the reference signal. When the peak of the reference signal exceeds that of the

carrier signal, it is over-modulation ( $M > 1$ ); when over-modulation occurs, the PWM system loses its control function in the area that exceeds the peak of the carrier signal.

The characteristics of the converter when the input voltage becomes unbalanced is compensated by dividing the reference modulation factor,  $M$ , by the unbalanced rate,  $k'$ . Therefore, if the unbalanced rate,  $k'$ , becomes smaller than the reference modulation factor,  $M$ , the modified modulation factor,  $M_{u,v,w}$ , becomes over-modulation. Since the reference modulation factor is determined by the value of the DC voltage as shown in Eq.(7), the higher the voltage is set, the narrower the compensation range becomes.

In order to enlarge the compensation range, we investigated a method of injecting the third harmonic into the reference signal; the waveforms are shown in [Fig. 4](#) when this is done: the waveform has a trapezoidal shape (reference signal other than  $s = 0$ ). The injection method uses the principle that the peak of the synthesized reference signal can be lowered by injecting the third harmonic into the reference signal.<sup>(6)</sup> When this method is used, the controlling capability can still be maintained in the over-modulation area because the peak of the reference signal does not exceed that of the carrier signal even though the reference modulation factor,  $M$ , becomes larger than 1.

Here, the reference signal,  $V_{rb}$ , can be expressed by the following equation when the third harmonic of amplitude,  $s$ , times that of the fundamental wave is injected.

$$V_{rb} = A \sin \omega t + sA \sin 3\omega t \quad \text{--- (13)}$$

where,  $A$ : amplitude of fundamental wave.

Next, the value of,  $s$ , that minimizes the peak value of the standard reference signal,  $V_{rb}$ , is calculated as follows. First, the standard reference signal,  $V_{rb}$ , is differentiated by time,  $t$ , to give the following equation:

$$\frac{dV_{rb}}{dt} = A(1 - 9s + 12s \cos^2 \omega t) \cos \omega t \quad \text{--- (14)}$$

From the above equation, the standard reference signal,  $V_{rb}$ , becomes the maximum value when  $\cos \omega t = \sqrt{\frac{9s-1}{12s}}$ . The maximum value of the standard reference signal,

$V_{rb}$ , is given by the following equation:

$$V_{rm} = \frac{A}{3} \sqrt{\frac{(3s+1)^3}{3s}} \quad \text{--- (15)}$$

The minimum peak value of the standard reference signal can be obtained by calculating the pole value of Eq.(15) as shown by following equation :

$$\frac{dV_{rm}}{ds} = \frac{A(6s-1)}{6s} \sqrt{\frac{(3s+1)}{3s}} \quad \text{--- (16)}$$

In other words, when  $s = 1/6$  (16.6%), the standard reference signal,  $V_{rb}$ , becomes the minimum so the peak value becomes  $0.866A$  to the amplitude,  $A$ , of the fundamental wave. Therefore, the apparent modulation factor,  $M'$ , can be linearly controlled within the range shown by the following equation, by increasing the peak value,  $V_{rm}$ , of the standard reference signal,  $V_{rb}$ , to the peak value of the carrier signal.

$$0 < M' < 1.13 \dots (17)$$

As the range that can be linearly controlled is widened by 1.13 times, the above equation shows that the compensation range of the converter characteristics for unbalanced input voltage is expanded by 1.13 times, and thus the output voltage can be improved by 13% by setting the apparent modulation factor to 1.13.

We could thus enlarge the compensation range by injecting the third harmonic, but the phase current becomes a trapezoidal waveform including the third harmonic because the third harmonic is injected into the reference signal. However, the third harmonic that exists in the phase current is canceled since the primary windings of the input transformer are connected in a delta connection, so the input line current has a sinusoidal waveform.<sup>(7)</sup> The actual measured waveform of input line current,  $i_u$ , and phase current,  $i_{pu}$ , of the U-phase are shown in Fig. 5. The phase current has a trapezoidal waveform since the third harmonic is injected, but the line current has a sinusoidal waveform because the third harmonic is canceled due to the delta connection of the transformer.

## 4. Simulation and Verification by Experiment

We confirmed the operation of our characteristics compensation circuit by simulation and experiment. A trial converter having output power of 10 [kW] was used for the experiment. The values shown in Table 1 were used as the conditions of the simulation and experiment.

**Table 1 Circuit conditions for simulation and experiment**

Item	Symbol	Value [unit]
Output Power	$P_o$	10[kW]
Input voltage	$E_{ac}$	200[V]
Output voltage	$E_{dc}$	250[V]
Input capacitor	$C_{ac}$	10[μF]
Output reactor	$L_{dc}$	1[mH]
Output capacitor	$C_{dc}$	10[mF]
Carrier frequency	$f_c$	15.6[kHz]

Fig. 6 shows the circuit configuration used to obtain the unbalanced input voltage in the experiment. Since the circuits of the respective phases are insulated from each other by the input transformer, the I.V.R. (Induction Voltage Regulator) can be inserted into the W-phase. The three-phase input voltage is made unbalanced by changing the I.V.R. in the W-phase.

The waveforms at various points when the characteristics compensation circuit is used are shown in Fig. 7. These are the waveforms when the unbalanced factor,  $k$ , is 10%. Here, the third harmonic is not injected. The input current,  $i_w$ , in Fig. 2 (b) is distorted, but the input current,  $i_w$ , shown in Fig. 7 (b) maintains the sinusoidal waveform and the power factor of almost 1 is maintained even though the input voltage,  $e_w$ , of the W-phase is decreased. The reactor current,  $i_L$ , in Fig. 2 (c) includes the low frequency component, but the reactor current,  $i_L$ , with compensation does not include the low frequency component as shown in Fig. 7 (c), but includes only the high frequency component caused by switching. This improvement applies to the DC voltage,  $V_{dc}$ , as shown in Fig. 7 (d), and the ripple

of low frequency component does not exist when the compensation circuit is used. The waveforms at various points when the characteristics compensation circuit is used and the unbalanced factor,  $k$ , is 20% ( $E_{acw} = 100$  [V]) are shown in Fig. 8. The reference signal,  $V_{rw}$ , in Fig. 7 (e) is not over-modulated (where peak value,  $V_{cm}$ , of the carrier signal is set to 10 [V]), but the reference signal,  $V_{rw}$ , of the W-phase is over-modulated as shown in Fig. 8 (e). As a result, the input current,  $i_w$ , is distorted at around its peak as shown in Fig. 8 (b). The low frequency component exists in the reactor current,  $i_L$ , and the low frequency ripple is included in the output voltage,  $V_{dc}$ .

Fig. 9 shows the relationship between the low frequency harmonic current,  $I_h$ , that is included in the reactor current,  $i_L$ , and the unbalanced factor,  $k$ . Comparing the measured value with the simulation value, the measured value agrees fairly well with the simulation value for the 120 and 240 [Hz] frequency components. Fig. 10 shows the relationship between the input current distortion factor,  $\sigma$ , and the unbalanced factor,  $k$ . Comparing the measured value with the simulation value, the measured value and the simulation value of the distortion factor,  $f_D$ , of the input current of each phase agree with each other. The characteristics start to deteriorate at around 15% unbalanced factor in Fig. 9 and Fig. 10. When the AC input voltage is set to 200 [V] and the DC voltage is set to 250 [V] on the balanced condition, the reference modulation factor becomes 0.58 as calculated from Eq.(7). The unbalanced factor becomes 16.3% when the unbalanced rate,  $k'$ , is 0.58 as derived from Eq.(8) and Eq.(11). Therefore, as shown in Fig. 3, overmodulation of the reference signal occurs when the unbalanced factor becomes 16.3% or more because the reference signal is divided by the unbalanced rate, which results in loss of linear control.

We have confirmed from simulation and experiment that the characteristics of the high power factor converter can be compensated by the proposed compensation circuit as long as the unbalance factor is 16.3% or less even though the three-phase input voltage is unbalanced.

The range that can be compensated by the compensation circuit is determined by the unbalanced rate,  $k'$ , and the reference modulation factor,  $M$ . As the setting of the DC voltage is increased, the compensation range becomes narrower, but it can be enlarged to 1.13 times by injecting the third harmonic into the reference signal.

## 5. Conclusion

In this paper, following points are clarified :

- (1) We proposed a characteristics compensation method using feed-forward control without using a current detector, for high power factor converters when the three-phase input voltage is unbalanced.
- (2) Using this compensation system, the modified reference signal,  $V_{rl}$ , is over-modulated and linear control cannot be performed when the unbalanced rate,  $k'$ , becomes smaller than the modified reference signal,  $V_{rl}$ . In other words, the compensation range is determined by the unbalanced rate,  $k'$ , and the reference modulation factor,  $M$ . As the setting of the DC voltage is increased, the compensation range becomes narrower, but it can be enlarged to 1.13 times by injecting the third harmonic into the reference signal.
- (3) We confirmed from our simulation and experiment that the characteristics of the input current and those of the DC output voltage do not deteriorate in the range where the modified reference signal is not over-modulated, even though the three-phase input voltage is unbalanced.

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Fig.1 Main circuit configuration of the three-phase high power factor converter with multiple-connected single-phase back type choppers

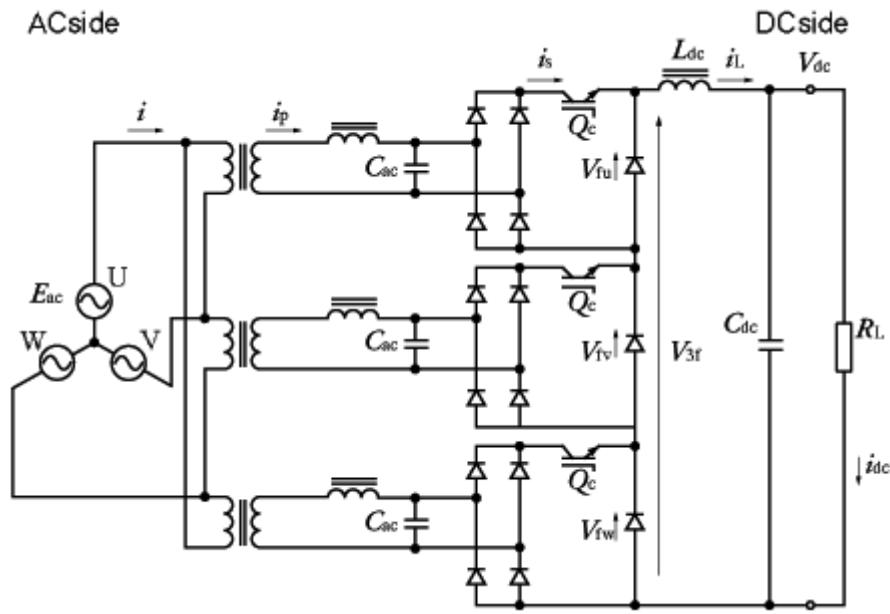


Fig.2 Actual waveforms measured at various points when the input voltage is unbalanced

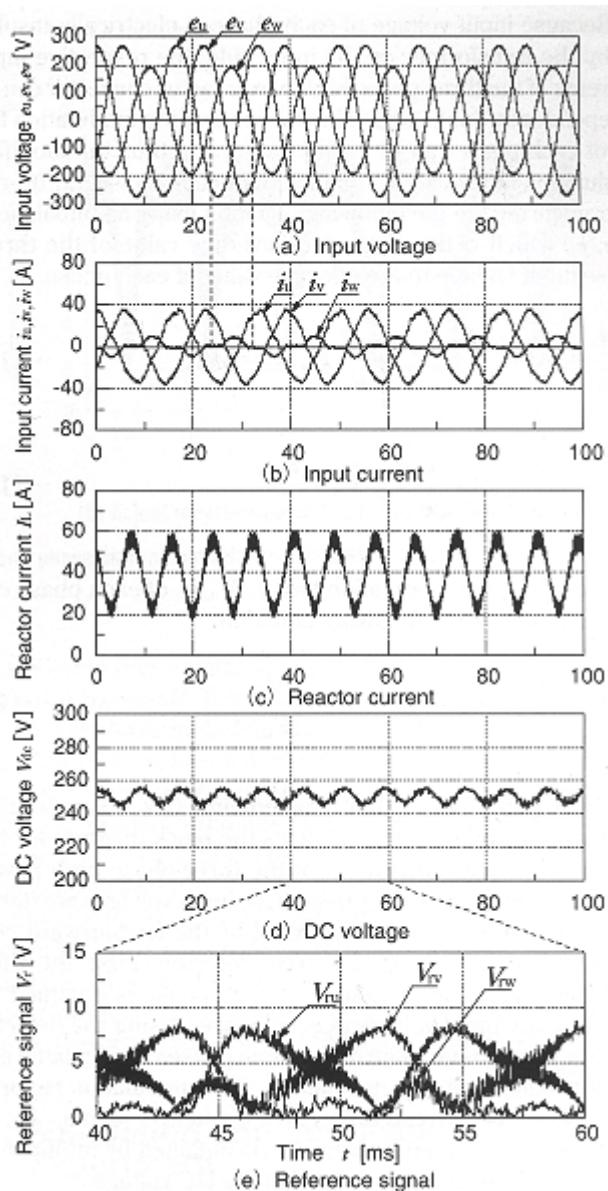


Fig.3 Characteristics compensation block diagram of three-phase high power factor converter for unbalanced three-phase AC input voltage

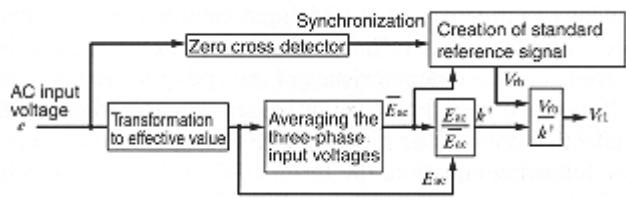


Fig.4 Waveform when third harmonic is injected into reference signal

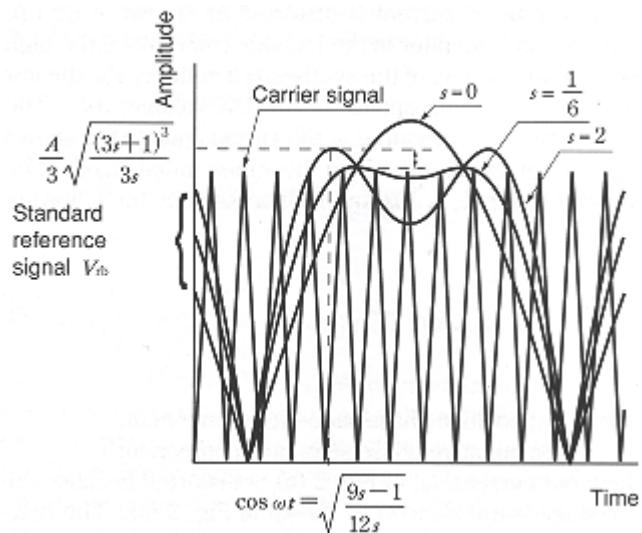
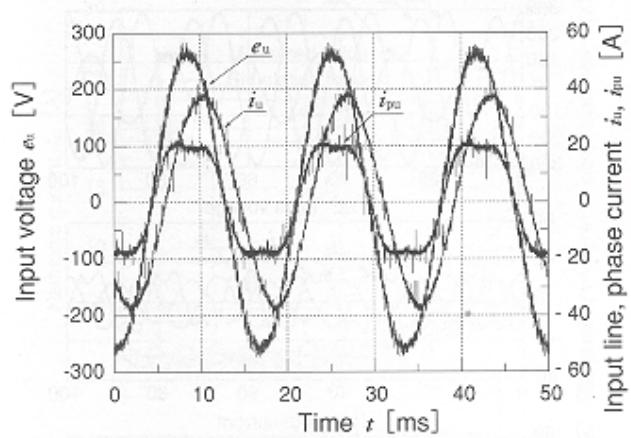
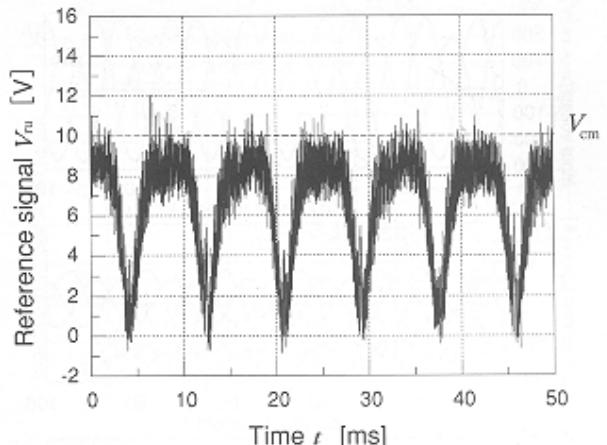


Fig.5 Actual measured waveform of input line current, phase current and reference signal



(a) Actual measured waveform of input line current and phase current



(b) Actual measured waveform of reference signal

Fig.6 Circuit configuration to obtain the unbalanced input voltage

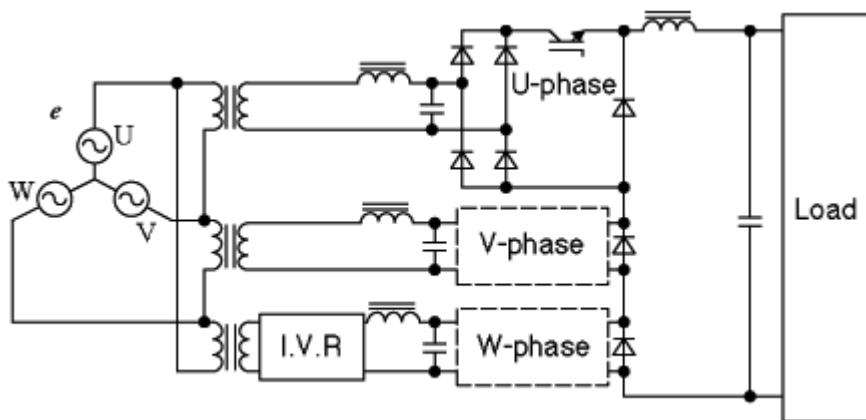


Fig.7 Waveforms at various points when the characteristics compensation circuit is used (Unbalanced factor k= 10%)

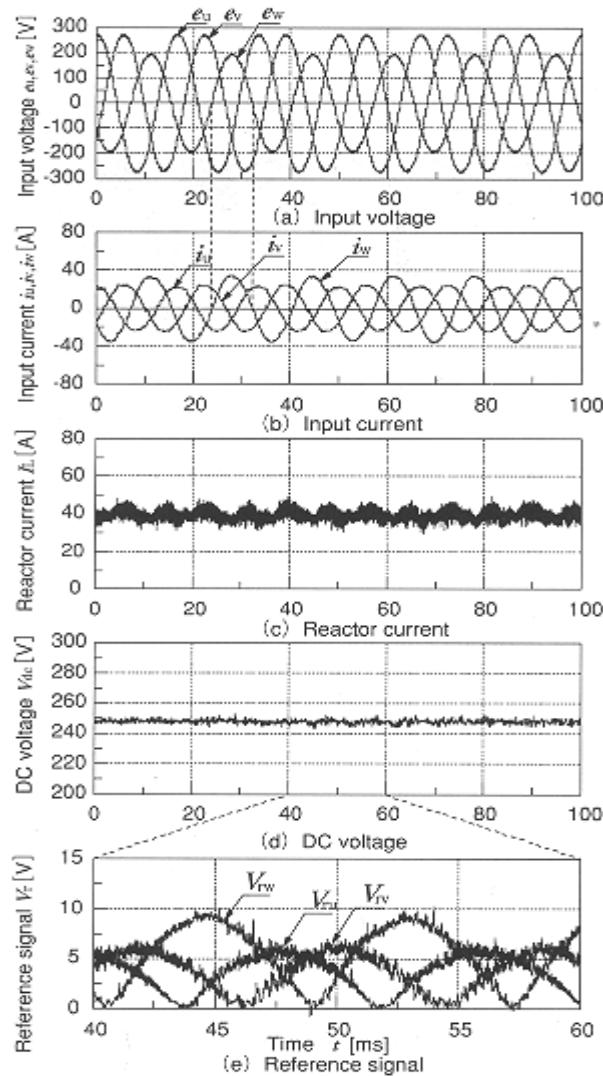


Fig. 8 Waveforms at various points when the characteristics compensation circuit is used (Unbalanced factor  $k = 20\%$ ,  $E_{ACW} = 100$  [V])

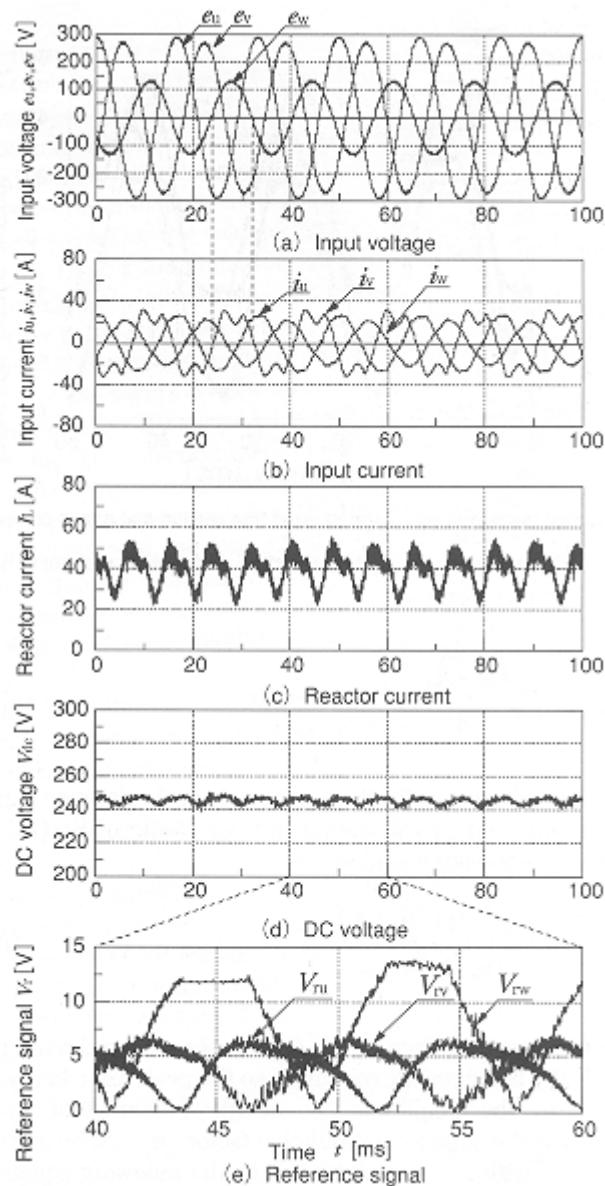


Fig. 9 Relationship between low frequency harmonic current  $I_h$  included in reactor current  $i_L$ , and unbalanced factor  $k$

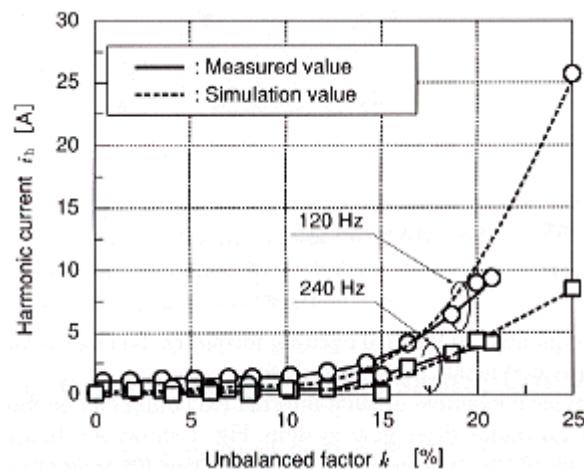


Fig. 10 Relationship between input current distortion factor  $\sigma$  and unbalanced factor k

